

of summability is shown to comprise an area bounded by a. (finite or infinite) number of straight lines, each of which goes through a critical point. This new region M. Borel calls the *polygon of summability*. An obvious question arises here; does the continuation of $f(x)$ obtained by exponential summation necessarily coincide with one obtained by other methods, for example Weierstrass's? In some cases it certainly does; for instance, when $f(x)$ is a rational function of x , or one branch of an algebraic function.

So far it has been supposed that the object of inquiry is in the first instance a series given by the law of construction of its terms; and the main result has been to show how, in certain cases where the series is divergent in the ordinary sense, it may be associated with a finite function, called its sum (in an extended sense), which the series so far represents that relations satisfied formally by the series are actually and arithmetically satisfied by its sum. But there is another side of the question which is of equal importance, especially from the practical point of view. We may have a function explicitly or implicitly defined by certain properties, and try to obtain a series which for purposes of computation or otherwise may be regarded as its equivalent. A typical illustration is afforded by the ordinary process of solving differential equations by series; here we have a uniform method which, if it does not fail altogether, leads us to a power-series, formally satisfying the equation, but not necessarily convergent. Exponential summation, when it is applicable, enables us to obtain a solution from the merely formal equivalent. In this connection we have Poincaré's theory of asymptotic series, which is expounded by M. Borel in Chapter i. Independently of its convergence, the expansion

$$c_0 + \frac{c_1}{x} + \frac{c_2}{x^2} + \dots + \frac{c_n}{x^n} + \dots$$

is said to represent $f(x)$ asymptotically if

$$x^n \left[f(x) - c_0 - \frac{c_1}{x} - \frac{c_2}{x^2} - \dots - \frac{c_n}{x^n} \right]$$

vanishes when x is infinite. Asymptotic expansions may be combined by the ordinary formal rules of rational operations, and the result is asymptotically equivalent to the corresponding combination of the functions represented. These considerations justify the use of semi-convergent series in computation; the classical example occurs in the theory of the gamma function. It must be carefully observed, however, that although the asymptotic expansion (if it exist) of a definite function is itself definite, we *cannot* infer the existence of a definite function corresponding to a given expansion $\sum c_n x^{-n}$; the reason of this rather paradoxical result is that innumerable functions (for instance e^{-x}) lead to an asymptotic expansion with zero coefficients throughout.

In Chapter ii. M. Borel discusses the results contained in Stieltjes's memoir (*Annales de la Faculté des Sciences de Toulouse*, tt. viii. ix.), and in Chapter v. deals with the polynomial expansions due to Mittag-Leffler. Interesting as they are, it seems hopeless to try to analyse these chapters within the compass of a review; they are, indeed, themselves of the nature of summaries, and will be best appreciated by those readers who accept M. Borel's invitation to consult the original memoirs. Attention may, however, be called to the author's

estimation of these researches. It is, in effect, that the memoir of Stieltjes, though of great originality and suggestiveness, is of restricted application and not likely to lead to a general theory; and that, on the other hand, while Mittag-Leffler's theory does not immediately afford a calculus of divergent series, in the proper sense of the term, it may very probably lead to one. It should be added that M. Borel himself has made substantial contributions to this theory of polynomial expansions; some of them [appear for the first time in the present volume.

The fact is that most of the field traversed in this very attractive course is of recent discovery, and we cannot expect to be presented with a complete and symmetrical doctrine all at once. Let us be thankful that M. Borel, himself one of the pioneers on this novel route, has so clearly and impartially indicated the progress that has hitherto been made.

G. B. M.

OUR BOOK SHELF.

The Chemical Essays of Charles-William Scheele. Pp. xxx + 294. (London: Scott, Greenwood and Co., 1901.) Price 5s. net.

THIS is a reprint of Dr. Beddoes' translation of Scheele's essays, which was published in 1786 by John Murray and may still be picked up occasionally in second-hand book shops. The reproduction is faithful even to the mis-spelling of Priestley's name in Beddoes' preface. Between this preface and the essays, however, there now appears a memoir of the life and work of Scheele, written for the reissue by Mr. John Geddes McIntosh. Mr. McIntosh presumably has inspired the reissue of the essays, and if this will be the means of getting them more generally read by students of chemistry, he may so far prove a benefactor.

Of the essays themselves it is hardly necessary to say anything. The facts they establish belong for the most part to what is now very elementary chemistry and the phlogistic hypothesis with which the explanations are involved did not long outlive Scheele; but the spirit which breathes in these essays and the method they inculcate can never grow commonplace or antiquated.

The strict fidelity to experiment, the rare sagacity, the scrupulous and minute observation and the extraordinary experimental skill combine to make Scheele a model for all time. When we add to this the pathos of his early struggles, the simplicity of his blameless life and the nobility of his untimely death, there can be no wonder that Scheele is reckoned a hero among chemists.

It cannot be said that the memoir which accompanies these essays is worthy of the subject. Mr. McIntosh has apparently considerable enthusiasm for the solid virtues of Scheele and for the material outcome of Scheele's discoveries, but he shows little critical insight or literary taste. Speaking, for example, of the discovery of chlorine, he says: "Let us now glance at the radical errors of the French school, the chief of whom was Berthollet, the man who was the first to make practical application of Scheele's discovery, and, as is usually the case with such men, they propound a theory of their own, so that some at least of the merit, if not all of the original discovery, may descend upon their own mantle."

The violence here done to Berthollet, to the rules of English composition, and to a time-honoured metaphor is very remarkable.

On the following page it is stated to be "a well-known fact at the present day" that the product of distilling fluorspar and sulphuric acid in a glass retort is gaseous hydrofluosilicic acid.

The essay concludes with a comparison of Scheele with Shakespeare, somewhat to the disparagement of the poet, and with a very comical lament that so few people ever visit either Scheele's birthplace, "if it still be in existence," or his shrine at Köping, "where he died and where presumably his remains are interred." One might even suspect that Mr. McIntosh had not been there. A. S.

Use-Inheritance; Illustrated by the Direction of Hair on the Bodies of Animals. By W. Kidd. 8vo. pp. 47. Illustrated. (London: A. and C. Black, 1901.) Price 2s. 6d. net.

EVERY naturalist who has studied the ungulate mammals must have been struck with the curious variation in the direction of the hair which occurs even in closely-allied species, and has probably been much puzzled to account for these differences. Why, for instance, do the hairs on the back of all the Asiatic buffaloes point towards the head and those of their African allies in the opposite direction? In the former animals, as in all analogous instances, a whorl (in this case on the haunches) marks the point where the change in the direction of the hairs from the normal backward slope occurs. In the work before us the author, although he has not attempted to give a reason for the variation in the hair-slope of closely allied species, has done good service by classifying these "whorls" and "featherings," as typified in the horse. He has also shown that these features always occur at spots where two or more muscles are acting against one another, as is well exemplified on the forehead of the horse. It is therefore suggested that the production of such whorls has a dynamical origin. It is noteworthy that while whorls and featherings are very commonly developed in short-haired mammals, they are either absent or rudimentary in those with long hair.

The main argument of the book is, however, connected with certain peculiarities of the hair-slope in man. In normal instances this slope on the back of the head and neck diverges obliquely from the middle line, somewhat after the simian fashion. But in a second, or "exceptional," type the direction is just the reverse of this. It is suggested that while the normal type has been directly inherited from simian ancestors, the exceptional type (which is considered to be an acquired one) has been derived from the female line. It is further shown than on the human back the direction of the hair-slope is quite different from that which obtains in all apes and monkeys. "This aberration of hair-slope I have suggested," writes Dr. Kidd, "to be produced by the habit which man has of spending about a third of his life, during sleep, in lying mostly on his side, and, for some millenniums at least, with some sort of rest for his head."

These peculiarities in the hair-slope of man and other animals, adds the author, are congenital and not due to selection; hence, unless originally created with the forms of life in which they occur, they must have been produced in their ancestors by use or habit. From this it follows that, if the creation hypothesis be discarded, in this particular instance, at any rate, acquired characters are inherited.

While claiming recognition for his own views on this point, Dr. Kidd (p. 8) deprecates the idea that they militate seriously against the merits of Weismann's theory as a whole. It may be added that the language in which this tribute to a great thinker is couched would have been more elegant had the author avoided the triple use of the word "which" in a single sentence. R. L.

Foundations of Botany. By Joseph Y. Bergen, A.M. Pp. x + 257. (Boston, U.S.A.: Ginn and Co., 1901.)

THIS book is intended, or at least so we gather from the preface, to provide a year's work in an advanced school class. Much of the matter is to be commended, and

some of the new figures are admirable. From this side the Atlantic one can only regard with envy the amount of energy expendable in American schools if a work of this proportion and scope is really suited to their possible requirements; for the book covers a wide range of subjects, and will make no small demands on the time of the student who aims at mastering its contents. The author clearly intends that the work shall be grappled with thoroughly, and from the concrete and practical side. He gives directions for laboratory work, and suggests problems to be solved by observation and experiment. These are incorporated in the text as appendices to the chapters, after the prevailing fashion in modern American text-books. It may, perhaps, be questioned whether the book might not be improved by the separation of the purely systematic portion into a volume by itself.

LETTERS TO THE EDITOR.

[The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts intended for this or any other part of NATURE. No notice is taken of anonymous communications.]

The Exploration of the Atmosphere over the Ocean.

THE experiment of flying kites in calm weather from the deck of a moving tug-boat, which was mentioned in NATURE of September 5 (p. 453), was continued by my assistant, Mr. Sweetland, and myself on a steamship that performed the voyage from Boston to Liverpool between August 28 and September 5. Flights were made on five days, the greatest height reached by the meteorograph being one-third of a mile, and the records of barometric pressure, air-temperature, relative humidity and wind-velocity, which are probably the first to be obtained above the North Atlantic, were shown to Section E of the British Association at Glasgow.

These experiments demonstrate conclusively that, with a steamer that can be manoeuvred at will, kites can be flown at sea in almost all weather conditions, and, consequently, a most important field is opened for their use in meteorological researches, especially in the tropics, where the conditions existing above the trade-winds are imperfectly known. It is to be hoped, therefore, that such an investigation will be undertaken either by the Government or by private enterprise, and I am now endeavouring to bring this about. A. LAWRENCE ROTCH.

Blue Hill Observatory, Hyde Park, Mass.,
U.S.A., October 25.

A Curious Flame.

THE kind of phenomenon described by Mr. Garbutt (p. 649) is frequently to be met with in "washed-out" flames, that is, in flames which are diluted to the point of extinction. In such cases the velocity of inflammation is so low that flame cannot propagate itself against the stream of gas. If the current of gas be baffled by an obstacle, then a flame may originate in the "slack waters" round the obstacle. No doubt the temperature of the obstacle is of some importance if the object be small, since rapid withdrawal of heat at any point of a gaseous current is a hindrance to the development of flame at that point. But it will be found that in the experiment described by Mr. Garbutt a flame may be obtained by opposing a large baffling surface of even an enduringly cold body such as a 56-pound weight. In this case a very large portion of the gaseous mixture is made stationary and the cooling down does not affect more than a small film next the metal. A kindred phenomenon is described in NATURE (vol. xlix. p. 86).

The flame obtained when a Bunsen lamp is lighted both at the bottom and the top of the tube is very feeble, and large tracts of it may be extinguished by holding in it beads of volatile salts. ARTHUR SMITHELLS.

November 4.

WITH regard to the flame described by Mr. Garbutt in your issue of October 31, I would suggest the following explanation, which, however, I have not as yet quite proved.

The products of the partial combustion at the bottom of the